Chapter 8

Pathfinder Networks and Multidimensional Spaces: Relative Strengths in Representing Strong Associates

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Multidimensional Scaling (MDS) (Kruskal, 1964) and the Pathfinder algorithm (Schvaneveldt, Durso, & Dearholt, 1985; Dearholt, Schvaneveldt, & Durso, 1985) share the goal of reducing large amounts of proximity data to an interpretable form. In psychological research, the resulting representations can reveal interesting relationships among concepts in memory. The two techniques, however, achieve their goals by way of different mechanisms. As a result, the two methods often produce very different representations of memory.

One goal of MDS is to represent the semantic dimensions underlying a domain of concepts. The dimensions revealed can provide information about how concepts are organized in memory. MDS positions points corresponding to the domain items into a space with *k*-dimensions. The euclidean distance between the points represents the psychological distance between the concepts.

Pathfinder uses a graph-theoretic technique which judges the importance of the relationships between items in each pair of concepts. Pathfinder produces a network representation of the concepts in a domain. It includes a link between two concepts in a network if and only if the link is a minimum length path between the two concepts. It defines a network which includes important links as indicated by the proximity data.

Both MDS and Pathfinder require estimates of psychological dissimilarity as input. These estimates are often obtained by having subjects rate the pair-wise relatedness of items in a domain. For example, subjects may rate the relatedness of pairs of animals on a scale of 1 to 5 (with 1 being extremely related and 5 being extremely unrelated). In doing so, subjects frequently find it easier to assign number ratings to related pairs than to unrelated pairs. For instance, subjects have no trouble determining that the pair *lion-tiger* should receive a rating of 1. They are very strong associates. Further subjects know that they are more related than say *lion-monkey* which may receive a rating of 2 or 3.

In contrast, subjects are often uncomfortable assigning numbers to unrelated items. These items are simply unrelated. It is difficult to give a number to this unrelatedness. Should the pair *whale-lion* receive a 4 or a 5? Is the pair more or less related than *elephant-penguin*? Assigning a number to the degree of unrelatedness often does not make sense. As a result, ratings for related pairs may contain more meaningful information than ratings for unrelated pairs. Further, subjects rate strong associates quickly, easily, and with a high degree of intersubject agreement. This indicates that these ratings may be particularly meaningful, as well as informative, about the structure of memory.

Although the ratings for strong associates may be particularly meaningful, MDS does not weight these ratings differently in determining the representation. This may cause some strong associates to be greatly distorted in MDS representations. MDS uses a least-squares technique in determining the arrangement of all concepts in some *k*-dimensional space. Each rating datum, whether it represents the relationship between *lion* and *tiger* or between *lion* and *trout*, exerts the same level of influence or constraint on the spatial solution. MDS tries to mutually satisfy these constraints by minimizing a least-squares measure called stress. Essentially, MDS may distort the representation of very psychologically meaningful strong associations to fit all ratings data, meaningful and otherwise, to some multidimensional space.

A paired-associates learning task was used in this study because it stresses relationships between individual pairs of concepts. Subjects are asked to recall the second word in a pair after being presented with the first word. The subject must form an association between the two words in order to be successful. Subjects learned lists of paired-associates which were linked in a Pathfinder solution or had very low interitem MDS distances. Lists were constructed using two sets of materials. One set was a relatively homogeneous domain which consisted of many strong associates. The other was a heterogeneous domain which had fewer strong associates. It was expected that many individual pairs would be distorted in the MDS space to achieve optimal fit of all concepts. This would cause some strongly associated items to be placed far away from each other in the MDS space. As a result, these strong associates would be excluded from the MDS list because of their large interitem distances. On the other hand, Pathfinder determines whether to link items on a pairwise basis. It has no global goodness-of-fit measure to optimize. As a result, these distortions should not take place in the Pathfinder solution. Linked items should be the strongest associates present in the materials.

The associates paired according to Pathfinder should be easier to learn than ones paired according to MDS solutions. Further, associates which are linked in Pathfinder and have small (highly related) link weights should be learned more easily than linked items with large link weights. This is simply because strong associates will have smaller weights due to smaller ratings values.

It was hypothesized that a paired-associates list organized according to the Pathfinder solution would be learned more quickly than one organized according to an MDS solution. Further, a Pathfinder list containing linked pairs with small link weights should be learned more easily than a similar list with larger link weights. However, all three of these lists should be learned more easily than a list of randomly selected word pairs.

Method

One group of subjects learned a list in which the word pairs had the closest interitem distances in the MDS solution (MDS group). Two other groups learned lists with items which were linked by the Pathfinder algorithm. One of the Pathfinder lists consisted of items with the smallest (i.e., most related) link weights in the Pathfinder output (Short-linked). The other had items which were linked in Pathfinder but had the largest link weights (Long-linked). The remaining condition served as a control, containing lists with items randomly selected (Random group) from the domains shown in Tables 1 and 2 at the beginning of each trial. Planned comparisons were performed. Performance on the Short-linked list was compared with that on the MDS list. Additionally, Short-linked was compared with Long-linked to determine if rating values (i.e., link weights) add information important for organization. Long-linked was compared to MDS to determine how the theoretically worst Pathfinder list compares to the best MDS list. Finally, the average of all structured lists was compared to the Random list to determine if semantic structure, in general, facilitates learning of paired-associates.

Additionally, many correlations among structural measures, as well as between structural and performance measures, were performed using the data collected from the Random group. Since random pairs were selected independently for each subject in the Random condition, data are available for many different word pairs. Any given word pair can be described by rating values, MDS interitem distances, Pathfinder links, and so on. Various correlations were performed to determine how predictive structural and ratings information is of paired-associate learning.

Construction of the Scaling Representations

Two sets of materials were used. Set I is fairly homogeneous, and includes a large number of strong associates. All of the items in this domain were animals of some type. The set is shown in Table 1. Set II consists of the materials used by Cooke, Durso, and Schvaneveldt (1986). This set, shown in Table 2, is more heterogeneous. It includes animals, plants, properties, parts of animals, parts of plants, and subordinate-superordinate relations. Further, it contains fewer strong associates than the materials in Set I.

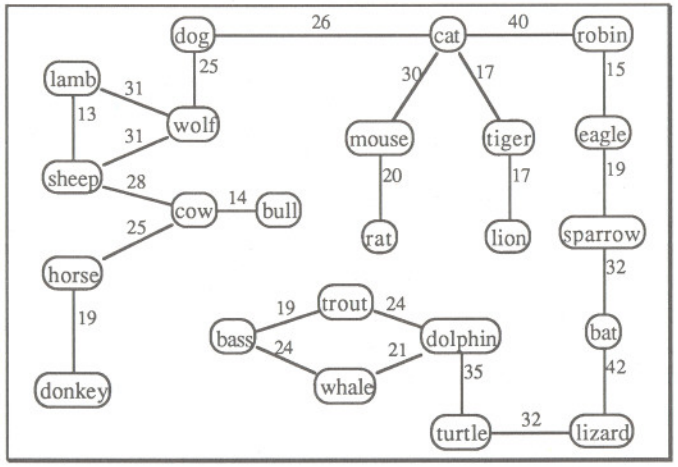

Figure 1. PFNET($r = \infty$, $q = n{-}1$) for the Set I materials.

Set I

Collection of Ratings. Fifteen students, enrolled in an introductory psychology course, rated the relatedness of all pair-wise combinations of animal concepts. Ratings were made on a scale from 0 to 5 with 5 being extremely related and 0 being unrelated. The concepts that were scaled are shown in Figures 1 and 2.

The scaling solution derived from these ratings was originally used for a different task in which subjects were asked to judge whether an X is an animal (see Cooke, Chapter 7, this volume). To make the set of items more homogeneous, ratings data of superordinates such as *fish, bird,* and *animal* were removed. There were 7 of these superordinates. Pathfinder and MDS representations were constructed using the remaining 23 basic-level concepts.

The interitem MDS distances and the structure of the Pathfinder solution may have been somewhat different if the superordinates were included. However, both scaling solutions were constructed from the same remaining data. There is no reason to believe that removing the superordinates had differential effects on Pathfinder and MDS solutions.

Scaling Solutions. The Pathfinder network for the Set I concepts is shown in Figure 1. This is the solution when $r = \infty$ and $q = n-1$, the sparsest network which can be derived. The assumption is that this network includes the most important links. The MDS solution for the concepts is shown in Figure 2. Three dimensions were chosen as the optimal dimensionality for the solution because stress and r^2 seemed to elbow at three dimensions. The Kruskal stress of the solution was .12.

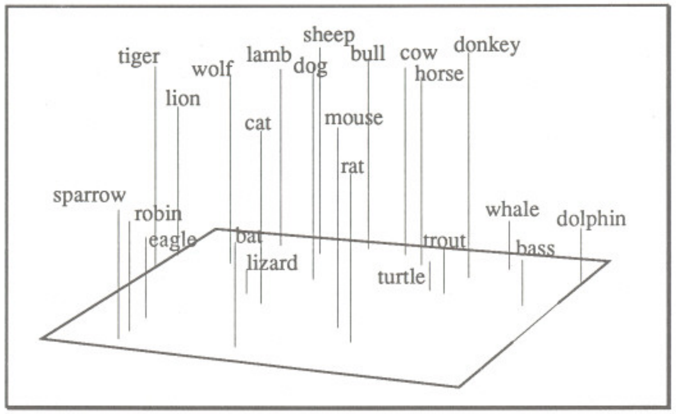

Figure 2. Three-dimensional MDS solution for the Set I materials.

Set II

Lists were also constructed from the materials and scaling solutions in Cooke et al. (1986). These materials are referred to as Set II and are shown in Figures 3 and 4.

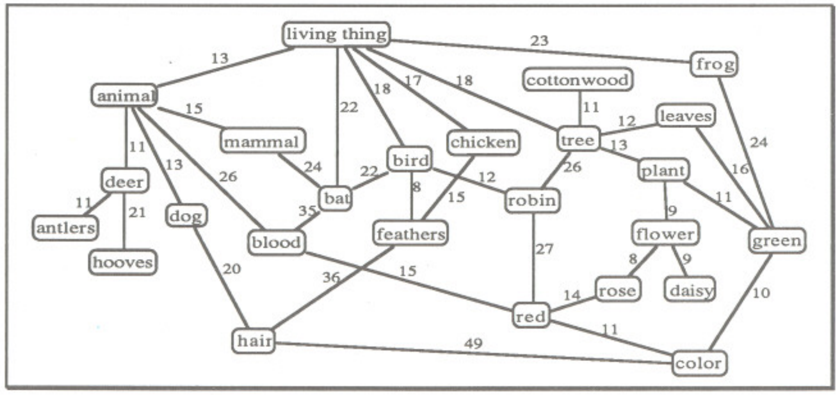

Figure 3. PFNET (parallel method) for the Set II materials.

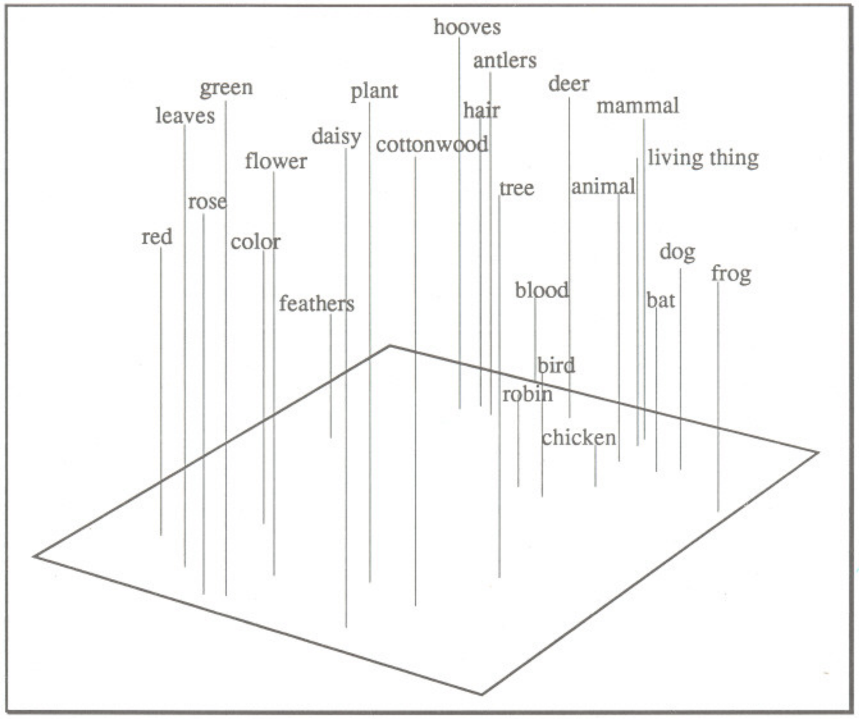

Figure 4. Three-dimensional MDS solution for the Set II materials. (From "Recall and Measures of Memory Organization," by Cooke, Durso, & Schvaneveldt, 1986, *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *12*(4), p. 542. Copyright 1986 by APA. Adapted by permission.)

Scaling Solutions. Cooke et al. (1986) used the parallel option of Pathfinder (Schvaneveldt et al., 1985) to construct their Pathfinder solution. The Pathfinder solution is shown in Figure 3. The ALSCAL-S (Young, Takane, & Lewyckyj, 1978) MDS solution (used in Cooke et al., 1986) for the materials is shown in Figure 4. Three dimensions were chosen as the optimal dimensionality for this because stress and r^2 seemed to elbow at three dimensions. The Kruskal stress for this solution was .21.

Construction of the Paired-Associate Lists

Selection of the items in each list was restricted in the following way. Any word appeared only once in a paired-associate list. The Short-linked list had those items which were linked by Pathfinder and had the smallest (i.e., most related) link weights. The Long-linked list had the linked items with the largest link weights. The MDS list contained those items with the smallest interitem distances. Random lists consisted of randomly selected word pairs chosen independently for each subject from the domains in Tables 1 and 2. The organized paired-associate lists, along with their mean interitem MDS distances, Pathfinder weights, and mean ratings, are shown in Tables 1 and 2.

Table 1. The paired-associates lists for the Set I materials.[a]

Linked Short	*Linked Long*	*MDS*
lamb-sheep	lizard-bat	cow-horse*
cow-bull	robin-cat	robin-eagle*
robin-eagle	dolphin-turtle	bull-sheep
tiger-lion	wolf-sheep	wolf-lion
donkey-horse	cow-horse	rat-mouse*
bass-trout	whale-bass	bass-trout*
rat-mouse	rat-mouse	whale-dolphin*
whale-dolphin	sparrow-eagle	sparrow-bat*
wolf-dog	tiger-lion	donkey-dog
MDS = 41.9	MDS = 61.6	MDS = 38.8
PF = 18.1	PF = 28.1	PF = N/A
Ratings = 18.1	Ratings = 28.1	Ratings = 26.2

[a]The mean PFNET weights, ratings, and interitem MDS distances are shown below the lists. A * next to an MDS pair indicates that it was also linked in the Pathfinder network.

Table 2. The paired-associates lists for the Set II materials.[a]

Linked Short	*Linked Long*	*MDS*
flower-rose	hair-color	bird-robin*
bird-feathers	blood-bat	mammal-deer
color-green	robin-red	tree-cottonwood*
tree-cottonwood	frog-green	antlers-hooves
animal-deer	living thing-flower	dog-bat
blood-red	deer-hooves	plant-daisy
living thing-chicken	feathers-chicken	color-red*
hair-dog	animal-mammal	flower-rose*
mammal-bat	plant-tree	leaves-green
MDS = 13.6	MDS = 20.3	MDS = 4.3
PF = 12.5	PF = 26.3	PF = N/A
Ratings = 12.5	Ratings = 26.3	Ratings = 19.3

[a]The mean PFNET weights, ratings, and interitem MDS distances are shown below the lists. A * next to an MDS pair indicates that it was also linked in the Pathfinder network.

An additional set of lists, which reversed the order of the word pairs in Set II, was constructed. As can be seen in Table 2, all of the pairs which contain superordinate-subordinate relationships list the superordinate first. However, it should not be assumed that superordinate-subordinate relationships are bidirectional. Reversing the order of these pairs provides information about whether this directionality is important.

Procedure

One hundred twenty-eight (80 in Set I, 48 in Set II) New Mexico State University undergraduate students participated in partial fulfillment of an experimental familiarity requirement. An additional 36 subjects learned the reversed Set II lists. Subjects were randomly assigned to one of eight combinations of materials and list type. Subjects were seated in front of a TERAK 8510 computer terminal and instructions which explained the procedure of the experiment were displayed on the screen. The subjects then received a familiarization trial, in which the items to be learned were shown on the screen one pair at a time for five seconds each. During the experimental trials, subjects were shown pairs of words in randomized order. The first item in the pair was presented on the screen and the subject's task was to type the appropriate accompanying word within 15 seconds. The correct answer was shown as soon as the subject typed the return key. The answer was displayed whether subjects made a correct or incorrect response or no response. There was no dropout method used. Subjects were encouraged to respond as quickly and accurately as possible. The task concluded when the entire list of word pairs was learned correctly or when the list had been presented 20 times.

Results

Planned comparisons (Keppel, 1982) on data collected using the Set I materials will be followed by those for Set II. These will be discussed in terms of three dependent variables: number of trials to achieve a 100% correct criterion, number of errors committed in achieving criterion, and reaction time to the first keypress on the final trial for each item. Also, correlations will be briefly discussed.

Set I

Means for the four conditions are shown in Table 3. As predicted, subjects in the Short-linked group learned the word associations in significantly fewer trials, $F(1,38) = 4.3$, $p < .05$, and with fewer errors, $F(1,38) = 8.4$, $p < .01$, than the subjects in the MDS group. Further, subjects in the Short-linked group performed better than those in the Long-linked group. Again, this is evidenced by fewer trials needed to reach criterion, $F(1,38) = 4.7$, $p < .05$, as well as fewer errors, $F(1,38) = 9.8$, $p < .01$. As expected, lists which were structured according to a scaling solution were easier to learn than the random lists. Subjects in the Random condition required more trials, $F(1,76) = 54.8$, $p < .001$, and made more errors, $F(1,76) = 52.02$, $p < .001$, than subjects who received structured lists. There were no significant differences on any measure between the Long-linked and MDS groups. Additionally, analysis of the RT data revealed no significant differences between any of the conditions.

Table 3. Means for the four conditions with the Set I materials.

List	*Trials*	*Errors*	*RT*
Short	1.7	.85	1726
Long	4.2	8.60	2019
MDS	4.0	7.80	1928
Random	7.7	25.30	2036

Set II

The means for subjects' performance are shown in Table 4. Again, analysis of planned comparisons indicates that subjects who learned a structured list did so in fewer trials, $F(1,44) = 12.6$, $p < .001$, with fewer errors, $F(1,44) = 24.5$, $p < .001$, and with faster reaction times, $F(1,44) = 7.8$, $p < .01$, than subjects who learned randomly arranged lists. All other comparisons were not significant. Further, there were no significant differences among groups with the reversed lists, or when data from the two lists were combined.

Table 4. Means for the four conditions with the Set II materials.

List	*Trials*	*Errors*	*RT*
Short	3.1	4.9	1817
Long	2.6	3.8	1704
MDS	3.3	6.3	1837
Random	4.9	5.3	2387

Correlations

The Random lists used in the experiment provide data about many individual pairs of items. These include performance data, such as number of errors made on a particular pair during the experiment, as well as structural data, such as the pair's interitem MDS distance and whether the pair was linked in Pathfinder. Various correlations between structural and performance measures, as well as among structural measures were performed on this data.

Several variations of Pathfinder networks were generated for exploratory purposes to determine if they are predictive of paired-associate learning. A more dense (i.e., contains more links) Pathfinder network, with $q = 2$ (PF2) was generated. Also, two matrices of graph-theoretic distance, one with $q = n{-}1$ (Graph) and one with $q = 2$ (Graph2) were generated. In these matrices each entry equals the number of links separating the two corresponding concepts in the Pathfinder network.

Most of the correlations between structural and performance measures were fairly low. It is probable that this is because each correlation tries to predict only one particular performance score given one piece of structural information. The correlations would probably be higher if we were trying to predict a mean of scores rather than one individual score. It should be mentioned that given the number of correlations performed, the family-wise error rate may be fairly high. Therefore, one should not put too much faith in any one correlation, since some correlations may be significant by chance alone.

There was little difference between the ability of Pathfinder and MDS to predict performance. Further, when the effects of the original ratings data were removed from the correlations, most of the resultant partial correlations were not significant. This indicates that correlations between structural measures and dependent measures could be accounted for by the ratings alone. The only exception to this was the Graph2 structural measure with the Set II materials.

Set I

Ratings correlated with percent correct, $r(178) = -.24$, $p < .05$, errors $r(178) = .2$, $p < .05$, reaction time, $r(178) = .17$, $p < .05$, and number of trials to the first correct trial, $r(178) = .19$, $p < .05$. MDS correlated with errors, $r(178) = .17$, $p < .05$, percent correct, $r(178) = -.24$, $p < .05$, and reaction time, $r(178) = .17$, $p < .05$. Pathfinder correlated with errors, $r(178) = -.19$, $p < .05$, percent correct, $r(178) = .24$, $p < .01$, and number of trials to first correct, $r(178) = -.19$, $p < .05$. Pathfinder with $q = 2$ (PF2) correlated with errors, $r(178) = -.18$, $p < .05$, percent correct, $r(178) = 2$, $p < .05$, and number of trials to first correct trial, $r(178) = -.16$, $p < .05$. Graph-theoretic distance with $q = n{-}1$ (Graph) was correlated with reaction time, $r(178) = .2$, $p < .05$. Finally, graph-theoretic distance with $q = 2$ (Graph2) correlated with percent correct, $r(178) = -.17$, $p < .05$, and reaction time, $r(178) = .17$, $p < .05$.

Set II

Ratings correlated with errors, $r(106) = .23$, $p < .05$, percent correct $r(106) = -.30$, $p < .01$, and number of trials to first correct trial, $r(106) = .27$, $p < .01$. MDS correlated with errors $r(106) = -.32$, $p < .01$. Finally, Graph2 correlated with errors $r(106) = .33$, $p < .01$, percent correct $r(106) = -.40$, $p < .01$, and number of trials to first correct trial, $r(106) = .38$, $p < .01$.

When the effects of ratings were removed from these correlations, almost all of the partial correlations between a structural measure and a performance measure were no longer significant. The only significant partial correlations involved Graph2 ($q = n{-}1$). Graph2 correlated with errors, $r(106) = .24$, $p < .01$, percent correct, $r(106) = -.27$, $p < .01$, and number of trials to first correct trial, $r(106) = .28$, $p < .01$.

Discussion

As expected, semantically structured lists were learned in fewer trials and with fewer errors than lists of random word pairs. However, the hypothesis that the Short-linked Pathfinder list would be learned more easily than the MDS list was confirmed only with materials from Set I. These were the animal concepts which included a large number of strong associates. With Set II, the domain with fewer strong associates, subjects performed equally well on the Pathfinder and MDS lists.

Whereas there were no differences among scaled lists in Set II, in Set I subjects required two and one-half times more trials to learn the MDS list than to learn the Short-linked Pathfinder list. Moreover, they made seven times as many errors while doing so. On average, subjects learned the Short-linked Pathfinder list in less than two trials, and they averaged less than one error. This means that many of the subjects recalled the list immediately after seeing the familiarization phase.

The ease with which this Short-linked list was learned indicates that the structure of the pairings is strongly related to the structure of the subjects' memory. Inspection of Table 1 shows how simple the Short-linked Pathfinder list is. Each first item is accompanied by a second item which is a strong associate. This is particularly the case with *tiger-lion*, *rat-mouse, wolf-dog*, and *lamb-sheep*. The least associated pair in the Short-linked group according to the ratings is *wolf-dog*. The MDS list is also easy, but it has fewer strongly associated pairs and more weakly associated pairs. Weak pairs include *donkey-dog, wolf-lion*, and *sparrow-bat*.

In Set I, Pathfinder seems to be linking primary (or at least strong) associates. Set I had more strong associates than Set II. Pathfinder links those items with the smallest ratings. If strongly associated items exist in the domain, they will be linked. Conversely, in a domain with few strong associates, links may not be between strongly associated concepts. They are simply between the most associated items in the domain. When this is the case, Pathfinder's ability to isolate strong associates may not be realized. On the other hand, because of the distortion of local relationships, MDS may be less effective at identifying strong associates.

It was predicted that the Short-linked Pathfinder would be learned more easily than the Long-linked Pathfinder. Again, this was true for the Set I materials but not the Set II materials. In Set I, word pairs with smaller link weights were easier to recall than word pairs with larger link weights. Again, this may be due to the reasons explained above. Effects due to differences in ratings may be most sensitive to strong associates.

Data from Humphreys and Greeno (1970) indicate that paired-associates learning consists of two main subprocesses: storage of the pairs in working memory and learning to retrieve these pairs. These subprocesses, they speculate, are analogous to recall and recognition memory. When subjects are shown the word pairs, they store them in memory using imagery, mnemonics, rehearsal, or some mediating word, and it is believed that the strategy for storing the pair will have a large effect on the nature of its representation in memory. Once stored in memory, its representation may not be optimal for retrieving the pair. Learning to retrieve the word pair is tantamount to fine tuning the memory structure so that it is easily accessed.

The effect of strong associates on the first stage of paired-associate learning is not clear. Perhaps strongly associated word pairs exist as pairs in memory prior to the task, whereas weakly associated pairs do not. It is easier to imagine how strong associates might affect the second stage. If the second stage is one of fine tuning the memorial representation of the items, this process is not needed (or is greatly abbreviated) for strong associates. That strong associates are spontaneously remembered indicates that the memorial representations of them are already quite suitable for access. So, the superiority of strong associates may be the result of eliminating the second stage of learning.

The present study indicates that both Pathfinder and MDS are good organizers of paired-associate learning material. This is evident in the superiority of these methods over random selection of items. However, when paired with previous results, this study suggests that Pathfinder is better than MDS at organizing materials for tasks which emphasize pair-wise relationships. This is qualified by the results which suggest that the presence or absence of strong associates may impact strongly on the relative superiority of Pathfinder. Nevertheless, Pathfinder is a better organizer of materials for a serial recall task, and a better predictor of performance on a free recall task (Cooke et al., 1986). Further, when strong associates are present, Pathfinder is a better organizer of materials for a paired-associates task.